Chapter 9

1

Linear Momentum and Collisions

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Lecture 04

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Center Of Mass

2

The Center of Mass

3

There is a special point in a system or object, called the center of mass, that moves as if all of the mass of the system is concentrated at that point.

The system will move as if an external force were applied to a single particle of mass M located at the center of mass.

- M is the total mass of the system.

This behavior is independent of other motion, such as rotation or vibration, or deformation of the system.

- This is the particle model.

3

Center of Mass, Coordinates

4

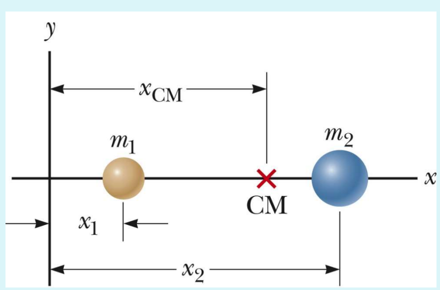
The coordinates of the center of mass are

$$X_{com}=\frac{\sum_i m_i x_i}{M}=\frac{m_1x_1+m_2x_2+\cdots}{m_1+m_2+\cdots}$$

$$Y_{com}=\frac{\sum_i m_i y_i}{M}=\frac{m_1y_1+m_2y_2+\cdots}{m_1+m_2+\cdots}$$

$$Z_{com}=\frac{\sum_i m_i z_i}{M}=\frac{m_1z_1+m_2z_2+\cdots}{m_1+m_2+\cdots}$$

M is the total mass of the system.



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Center of Mass, position

5

For a system of particles, the center of mass in three dimensions can be located by its position vector, $\vec{r}_{com}$.

- $\vec{r}_{com} = X_{com}\hat{\imath} + Y_{com}\hat{\jmath} + Z_{com}\hat{k}$

5

Motion of a System of Particles

6

- Assume the total mass, M, of the system remains constant.
- We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system.
- We can also describe the momentum of the system and Newton's Second Law for the system.

6

Velocity and Momentum of a System of Particles

7

The velocity of the center of mass of a system of particles is:

- $\vec{v}_{com} = \frac{\sum_i m_i \vec{v}_i}{M} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$

The momentum can be expressed as:

- $M\vec{v}_{com} = \sum_i m_i \vec{v}_i = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots = \vec{p}_{tot}$

The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass.

Acceleration and Force in a System of Particles

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The acceleration of the center of mass can be found by differentiating the velocity with respect to time.

- $\vec{a}_{com} = \frac{\sum_i m_i \vec{a}_i}{M} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots}{m_1 + m_2 + \cdots}$

The acceleration can be related to a force.

- $M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots = \vec{F}_1 + \vec{F}_2 + \cdots$

If we sum over all the internal force vectors, they cancel in pairs and the net force on the system is caused only by the external forces.

Newton's Second Law for a System of Particles

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- Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

$$\sum \vec{F}_{ext} = M\vec{a}_{com}$$

- The center of mass of a system of particles of combined mass M moves like an equivalent particle of mass M would move under the influence of the net external force on the system.

Impulse and Momentum of a System of Particles

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- The impulse imparted to the system by external forces is

$$\int \sum \vec{F}_{ext}\, dt = M \int d\vec{v}_{com}$$

$$\Delta\vec{p}_{tot} = \vec{I}$$

- The total linear momentum of a system of particles is conserved if no net external force is acting on the system.

$$M\vec{v}_{com} = \vec{p}_{total} = constant\ when \sum \vec{F}_{ext} = 0$$

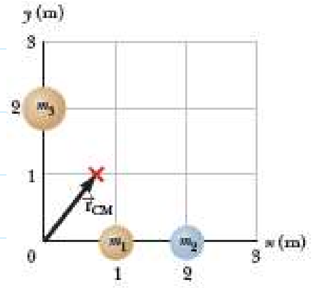
- For an isolated system of particles, both the total momentum and the velocity of the center of mass are constant in time.
  - This is a generalization of the isolated system (momentum) model for a many-particle system.

The com of Three Particles

Saturday, 30 January, 2021 15:30

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A system consists of three particles located as shown. Find the center of mass of the system. The masses of the particles are $m_1 = m_2 = 1\ kg$ and $m_3 = 2\ kg$.



Use the defining equations for the coordinates of the center of mass and notice that $z_{CM} = 0$:

$$x_{CM} = \frac{1}{M}\sum_i m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(1.0\text{ kg})(1.0\text{ m}) + (1.0\text{ kg})(2.0\text{ m}) + (2.0\text{ kg})(0)}{1.0\text{ kg} + 1.0\text{ kg} + 2.0\text{ kg}} = \frac{3.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 0.75\text{ m}$$

$$y_{CM} = \frac{1}{M}\sum_i m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(1.0\text{ kg})(0) + (1.0\text{ kg})(0) + (2.0\text{ kg})(2.0\text{ m})}{4.0\text{ kg}} = \frac{4.0\text{ kg}\cdot\text{m}}{4.0\text{ kg}} = 1.0\text{ m}$$

Write the position vector of the center of mass:

$$\vec{\mathbf{r}}_{CM} \equiv x_{CM}\hat{\mathbf{i}} + y_{CM}\hat{\mathbf{j}} = (0.75\hat{\mathbf{i}} + 1.0\hat{\mathbf{j}})\text{ m}$$

The Exploding Rocket

Saturday, 30 January, 2021 15:31

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A rocket is fired vertically upward. At the instant it reaches an altitude of $1000\,m$ and a speed of $v_i = 300\ m/s$ it explodes into three fragments having equal mass. One fragment moves upward with a speed of $v_1 = 450\ m/s$ following the explosion. The second fragment has a speed of $v_2 = 240\ m/s$ and is moving east right after the explosion. What is the velocity of the third fragment immediately after the explosion?

Use the isolated system (momentum) model to equate the initial and final momenta of the system and express the momenta in terms of masses and velocities:

$$\Delta \vec{\mathbf{p}} = 0 \quad \rightarrow \quad \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f \quad \rightarrow \quad M\vec{\mathbf{v}}_i = \frac{M}{3}\vec{\mathbf{v}}_1 + \frac{M}{3}\vec{\mathbf{v}}_2 + \frac{M}{3}\vec{\mathbf{v}}_3$$

Solve for $\vec{\mathbf{v}}_3$:

$$\vec{\mathbf{v}}_3 = 3\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2$$

Substitute the numerical values:

$$\vec{\mathbf{v}}_3 = 3(300\hat{\mathbf{j}}\ \mathrm{m/s}) - (450\hat{\mathbf{j}}\ \mathrm{m/s}) - (240\hat{\mathbf{i}}\ \mathrm{m/s}) = \boxed{(-240\hat{\mathbf{i}} + 450\hat{\mathbf{j}})\ \mathrm{m/s}}$$

Acceleration of com of three particles

Saturday, 30 January, 2021 15:32

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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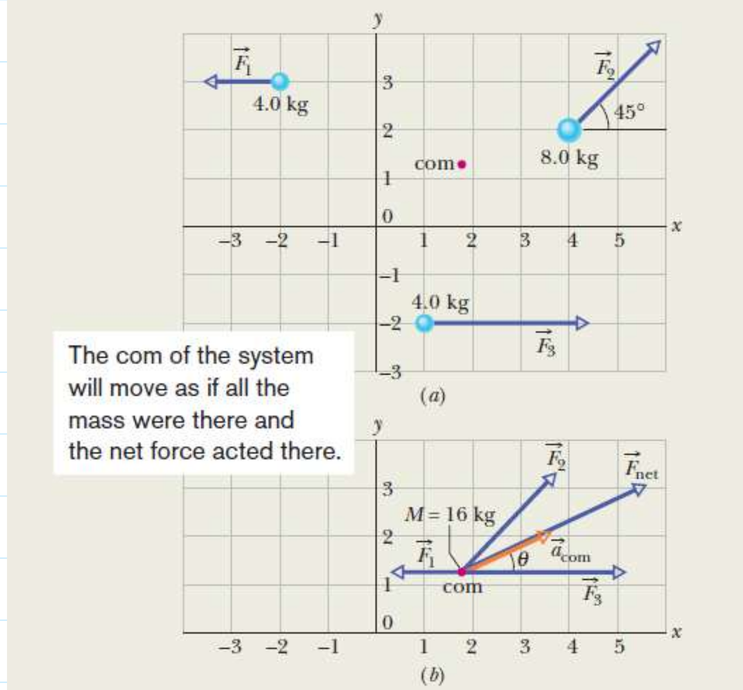
Three particles are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The three forces are: $\vec{F}_1 = 6\,N, 180^\circ$, $\vec{F}_2 = 12\,N, 45^\circ$ and $\vec{F}_3 = 14\,N, 0^\circ$.What is the acceleration of the center of mass of the system, and in what direction does it move?



Calculations: We can now apply Newton's second law ($\vec{F}_{net} = m\vec{a}$) to the center of mass, writing

$$\vec{F}_{net} = M\vec{a}_{com} \tag{9-20}$$

or

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

so

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}. \tag{9-21}$$

Equation 9-20 tells us that the acceleration $\vec{a}_{com}$ of the center of mass is in the same direction as the net external force $\vec{F}_{net}$ on the system (Fig. 9-7*b*). Because the particles are initially at rest, the center of mass must also be at rest. As the center of mass then begins to accelerate, it must move off in the common direction of $\vec{a}_{com}$ and $\vec{F}_{net}$.

We can evaluate the right side of Eq. 9-21 directly on a vector-capable calculator, or we can rewrite Eq. 9-21 in component form, find the components of $\vec{a}_{com}$, and then find $\vec{a}_{com}$. Along the x axis, we have

$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M} = \frac{-6.0\text{ N} + (12\text{ N})\cos 45^\circ + 14\text{ N}}{16\text{ kg}} = 1.03\text{ m/s}^2.$$

Along the y axis, we have

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M} = \frac{0 + (12\text{ N})\sin 45^\circ + 0}{16\text{ kg}} = 0.530\text{ m/s}^2.$$

From these components, we find that $\vec{a}_{com}$ has the magnitude

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2} = 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2 \qquad \text{(Answer)}$$

and the angle (from the positive direction of the x axis)

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27^\circ. \qquad \text{(Answer)}$$